AdS/CFT: Then and Now

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On June 30, 1997...

• A group of string theorists working on D-brane/black hole and D-brane/black brane correspondence. Polchinski; Strominger, Vafa; Callan, Maldacena; ...

• A stack of N Dirichlet 3-branes realizes $\mathcal{N}=4$ supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved RR charged background of type IIB theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left( -(dx^0)^2 + (dx^i)^2 \right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$
• Matching the brane tensions gives \[ L^4 = g_{YM}^2 N \alpha' \]

• In addition to the ‘t Hooft large N limit, a new dramatic simplification for \( g_{YM}^2 N \gg 1 \): the metric has small curvature everywhere.

• Bekenstein-Hawking entropy of near-extreme 3-brane

\[
S_{BH} = \frac{2\pi A_h}{\kappa^2} = \frac{\pi^2}{2} N^2 V_3 T^3
\]

• Agrees, up to a factor of 3/4, with that in weakly coupled SYM theory. Gubser, IK, Peet

• Low-energy absorption cross-sections agree exactly

\[
\sigma_{SUGRA} = \frac{\pi^4}{8} \omega^3 L^8 = \frac{\kappa^2 \omega^3 N^2}{32\pi}
\]
The AdS/CFT Duality
Maldacena; Gubser, IK, Polyakov; Witten

- The low-energy limit taken directly in the geometry. Maldacena
- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}=4$ SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS$_5$ space realizes the conformal symmetry of the gauge theory.
- Allows us to “solve” strongly coupled gauge theories, e.g. find operator dimensions $\Delta_{\pm} = 2 \pm \sqrt{4 + m^2 L^2}$
Three Lessons Learned

• Lesson 1: String theory can make definite, testable predictions!

• The dimensions of unprotected operators, which are dual to massive string states, grow at strong coupling as

$$2 \left( n g_{\text{YM}} \sqrt{N} \right)^{1/2}$$

• Verified for the Konishi operator dual to the lightest massive string state (n=1) using the exact integrability of the planar $\mathcal{N}=4$ SYM theory. Gromov, Kazakov, Vieira; ...

• Similar successes for the dimensions of high-spin operators, which are dual to spinning strings in AdS space.
Lesson 2: Color Confinement

- The quark anti-quark potential is linear at large distance but nearly Coulombic at small distance.

- The 5-d metric should have a warped form by Polyakov

\[ ds^2 = \frac{dz^2}{z^2} + a^2(z)(- (dx^0)^2 + (dx^4)^2) \]

- The space ends at a maximum value of \( z \) where the warp factor is finite. Then the confining string tension is

\[ \frac{a^2(z_{max})}{2\pi \alpha'} \]
In some models, like the warped deformed conifold, the confinement happens dynamically through dimensional transmutation. \textsuperscript{IK, Strassler}

\[ ds_{10}^2 = h^{-1/2}(y) \left( - (dx^0)^2 + (dx^i)^2 \right) + h^{1/2}(y) ds_6^2 \]
\[ \sum_{i=1}^4 z_i^2 = \varepsilon^2 \]

However, the string dual of asymptotically free gauge theory remains elusive.
• Lesson 3: The whole thing has become WAY more than anyone expected 20 years ago.

• I am amazed by the range of applications of the gauge/gravity duality.

• In addition to the strongly coupled plasmas and many body physics, we have learned a lot about quantum entanglement and quantum information. Maldacena; Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi; IK, Kutasov, Murugan; Myers, ...

• This is teaching us a lot about the mysteries of black holes and quantum gravity.

• We have also learned a great deal about the Chern-Simons matter CFTs using both the ABJM type models and the higher-spin AdS/CFT. Aharony et al; Giombi et al; ...
A Brief Wish List

• Getting better control over the regime where the coupling is not very large, but is of order 1. Most gauge theories, including the non-supersymmetric ones, are in this regime. This is crucial for understanding the large N QCD more quantitatively.

• A better understanding of the 1/N corrections to observables. This is crucial for the applications to quantum gravity.

• CFTs dual to de Sitter space.
Another Wish: More Melons

- The “melonic” large N limits, which appear in the tensor models, have already been connected with SYK-like models. Gurau; Witten; IK, Tarnopolsky;...

- Hopefully, the tensor models will find other uses.

\[ g^2N^6 \sim N^3\lambda^2 \]

\[ g^4N^9 \sim N^3\lambda^4 \]
• HAPPY 20\textsuperscript{TH} BIRTHDAY, ADS/CFT!
• AND MANY HAPPY RETURNS!
D-Branes vs. Geometry

- Dirichlet branes led string theory back to gauge theory in the mid-90’s. Polchinski

- A stack of N Dirichlet 3-branes realizes $\mathcal{N}=4$ supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(- (dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2)$$

which for small $r$ approaches $AdS_5 \times S^5$

whose radius is related to the coupling by$$L^4 = g_{YM}^2 N \alpha'^2$$
• Gauge invariant operators in the CFT\(_4\) are in one-to-one correspondence with fields (or extended objects) in AdS\(_5\).

• Operator dimensions are an important set of quantities

\[ \langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2) \rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x_1 - x_2|^{2\Delta_1}} \]

• The operator dimension is related to mass of the corresponding field in AdS space:

\[ \Delta_\pm = 2 \pm \sqrt{4 + m^2L^2} \]
The quark anti-quark potential

• The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.

• The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential

\[ V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r} \]

Maldacena; Rey, Yee
Confinement and Warped Throat

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

\[ ds_{10}^2 = h^{-1/2}(y) \left( - (dx^0)^2 + (dx^i)^2 \right) + h^{1/2}(y) ds_6^2 \]

- \( ds_6^2 \) is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

\[ \sum_{i=1}^{4} z_i^2 = \varepsilon^2 \]
• The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

• The dual gravity provides a `hyperbolic cow’ approximation, i.e. a toy model, for QCD.
Confinement and Entanglement

• Due to the confinement, there is a phase transition in the behavior of the entanglement entropy as function of the strip width. IK, Kutasov, Murugan; Nishioka, Takayanagi

• There is evidence for a similar transition or crossover in lattice gauge theory. Velitsky; Buividovich, Polikarpov; Nakagawa, Nakamura, Motoki, Zakharov
Desperately Seeking non-SUSY

- What are the solid non-supersymmetric examples of the AdS/CFT correspondence?
- Some initial hopes for a large class of non-supersymmetric CFTs at large N, which are certain truncations of the maximally supersymmetric one, were dashed by explicit calculations at small ‘t Hooft coupling $\lambda$.

IK, Dymarsky, Roiban
Consider gauge theories on a stack of D3-branes at the tip of a cone \(\mathbb{R}^6/\Gamma\) where the orbifold group \(\Gamma\) breaks all the supersymmetry.

At first sight, the large N gauge theory seems conformal because the beta functions for all single-trace operators vanish. The candidate string dual is \(\text{AdS}_5 \times S^5/\Gamma\). Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson

However, dimension 4 double-trace operators made out of twisted single-trace ones, \(f O_n O_{-n}\), are induced. Their one-loop planar beta-functions have the Coleman-Weinberg form

\[
\beta_f = a \lambda^2 + 2 \gamma f \lambda + f^2
\]
\[
\beta_\lambda = 0
\]

In any non-SUSY orbifold theory there are some such beta functions that don’t have real zeros.

To complement this perturbative non-SUSY problem, the dual gravitational backgrounds were found to have non-perturbative instabilities. Horowitz, Orgera, Polchinski
AdS$_4$/CFT$_3$

• Besides describing all known particle physics, Quantum Field Theory is important for understanding the vicinity of certain second order phase transitions, such as the all-important water/vapor transition.

• This transition is in the 3-dimensional Ising Model Universality Class.
Critical O(N) Model

- Describes 2\textsuperscript{nd} order phase transitions in statistical systems with O(N) symmetry.

\[ S = \int d^3x \left[ \frac{1}{2} (\partial_\mu \phi^a)^2 + \frac{\lambda}{2N} (\phi^a \phi^a)^2 \right] \]

- Can be studied using the Wilson-Fisher expansion in \( \varepsilon=4-d \).

- The model simplifies in the large N limit, where it has approximately conserved currents of any even spin.
Higher Spin AdS/CFT

• An AdS\(_4\) dual of the large N model has been proposed. IK, Polyakov

• It is the Vasiliev theory of an infinite number of interacting massless higher-spin gauge fields including gravity.

• Seems simpler than string theory; has only one Regge trajectory.

• There is no small AdS curvature limit. Yet, a great deal of recent progress producing agreement between the Vasiliev theory and the d=3 O(N) model. Giombi, Yin; Giombi, IK

• This class of examples of AdS/CFT does not rely on supersymmetry.
Dualities among Chern-Simons Theories

- Higher spin AdS/CFT correspondence applies also to theories of massless fermions or bosons interacting with Chern-Simons gauge fields in 2+1 dimensions. Aharony et al.; Giombi et al.

\[ S_{CS} = \int d^3 x \epsilon^{\mu\nu\rho} \text{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) \]

- This served as motivation for conjecturing a precise duality between 2+1 dimensional bosonic and fermionic CFTs. Aharony, Gur-Ari, Yacoby

- Recently conjectured to apply not only at large N, but at small N as well, including U(1) gauge theories. Aharony; Seiberg, Senthil, Wang, Witten; Karch, Tong

- Very nice interplay with condensed matter physics, where Chern-Simons Theories play an important role.
Conclusions

• Throughout its history, string theory has been intertwined with the theory of strong interactions.
• The Anti-de Sitter/Conformal Field Theory correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories. In particular, allows to study Quantum Entanglement Entropy.
• Extensions of AdS/CFT provide a new geometrical understanding of color confinement and other strong coupling phenomena.
• d-dimensional CFT’s with dynamical fields in the fundamental representation of $O(N)$ dual to interacting higher-spin theories in $\text{AdS}_{d+1}$. 