Microstate Geometries

Deep Inside the Black-Hole Regime

Research supported in part by DOE grant DE-SC0011687
Outline

• Microstate Geometries D1-D5-P Holography
• Some families of D1-D5-P states
• Building the holographic duals: Microstate geometries with AdS$_2$/BTZ throats
• The MSW string
• Holographic duals of some MSW states

Based on Collaborations with:

Microstate Geometry Program

*Microstate Geometry* $\equiv$ Smooth, horizonless solutions to the *bosonic* sector of *supergravity* with the same asymptotic structure as a given black hole/ring.

*Singularity resolved; Horizon removed*
Microstate Geometry Program

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*Supergravity* because we seek stringy resolutions at the horizon scale

- Very long-range effects $\Rightarrow$ Massless limit of strings …
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*What is the form of generic, BPS, time-independent horizonless, smooth solutions in supergravity?*
Microstate Geometry Program

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What is the form of generic, BPS, *time-independent* horizonless, smooth solutions in supergravity?

What CFT states do they describe?
Primary Motivation for Microstate Geometries

Resolving the black-hole information problem seems to require microstate structure to be encoded and supported at the horizon scale.

Microstate Geometries

- The only (known) mechanism that can support structure at the horizon scale
- Supergravity captures the universal, macroscopic features of microstate structure
- Semi-classical analysis: To what extent can supergravity encode microstate structure?
Black-Hole Microstates and CFT’s
Black-Hole Microstates and CFT’s

- **D1-D5 CFT**: A (4,4) supersymmetric CFT with $c = 6 N_1 N_5$

  \[
  \frac{1}{4} \text{ BPS states} = (R,R)-\text{ground states}
  \]

  \[
  \frac{1}{8} \text{ BPS states} = (\text{any left-moving state}, \text{R ground state}) ^{N_P}
  \]

  **Strominger-Vafa state counting for BPS black hole in five dimensions**:
  \[
  S = 2\pi \sqrt{N_1 N_5 N_P}
  \]
Black-Hole Microstates and CFT’s

- **D1-D5 CFT:** A (4,4) supersymmetric CFT with $c = 6 N_1 N_5$
  
  $\frac{1}{4}$ BPS states = (R,R)-ground states
  
  $\frac{1}{8}$ BPS states = (any left-moving state, R ground state)

  Strominger-Vafa state counting for BPS black hole in *five dimensions*:
  
  $$ S = 2\pi \sqrt{N_1 N_5 N_P} $$

- **MSW String:** A (0,4) supersymmetric CFT *(Maldacena-Strominger-Witten)*
  
  M5 brane wrapping a divisor in a $\text{CY}_3$. Dual class, $P \in H^2(\text{CY}_3, \mathbb{Z})$
  
  MSW string CFT lives on remaining (1+1) dimensions of M5 brane
  
  Central charge $c = 6 D$,  
  
  $D = \frac{1}{6} \int_{\text{CY}_3} P^3$

  State counting for BPS black hole in *four dimensions*:
  
  $$ S = 2\pi \sqrt{D N_P} $$
One Focus of the Microstate Geometry Program

Describe the strongly coupled gravity duals of these CFT states. To what extent can these CFT states be captured in supergravity?

⇒ Universal gravity dual of both D1-D5 and MSW.
The D1-D5 CFT

Open D1-D5 superstrings moving in $T^4$ with $N = N_1 N_5$ Chan-Paton labels: $(T^4)^N / S_N$

$\Rightarrow$ CFT on common D1-D5 direction, $(t,y) \leftrightarrow (u,v)$

$(4,4)$ supersymmetric CFT with $c = 6 N_1 N_5$
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$y = y + 2\pi R$

Maximally spinning ($\frac{1}{4}$ BPS) RR-ground state:
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Maximally spinning ($\frac{1}{4}$ BPS) RR-ground state:

$\begin{align*}
\mathbf{N} &= N_1 N_5 \\
\text{copies} & \\
(+) &= (+,+) \\
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\end{align*}$

$\begin{align*}
(\mathbf{j}_L, \mathbf{j}_R) &= \frac{1}{2} (N, N)
\end{align*}$
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- Space-time angular momenta
  - $(+,+)$
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- Space-time angular momenta
  - $(j_L, j_R) = \frac{1}{2}(N, N)$

Holographic dual: Maximally spinning supertube in $R^{4,1}$

Supertube profile spins out into $M^{4,1}$ space-time

$$(g_1(v), g_2(v), g_3(v), g_4(v)) \in \mathbb{R}^4$$

$$g_1(v) + ig_2(v) = a e^{2\pi iv/R}$$

$$g_3(v) = g_4(v) = 0$$
The D1-D5 CFT

Open D1-D5 superstrings moving in $T^4$ with $N \equiv N_1 N_5$ Chan-Paton labels: $(T^4)^N/S_N$

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Maximally spinning (1/4 BPS) RR-ground state:

$\{ (+,+), (+,+), (+,+), (+,+), \}$

space-time angular momenta

$N = N_1 N_5$

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Holographic dual: Maximally spinning supertube in $R^{4,1}$

Supertube profile spins out into $M^{4,1}$ space-time

$Q_1 Q_5 = R^2 a^2$

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g_3(v) = g_4(v) &= 0
\end{align*}$

back-react

$\text{AdS}_3 \times S^3 \times T^4$
More general $\frac{1}{4}$ BPS profiles

Orbifold CFT: $k$ twisted sector

$k$ loops

$|+\frac{1}{2}, +\frac{1}{2}\rangle^k$  $\rightarrow$  Length $k$ loop

$|+\frac{1}{2}, +\frac{1}{2}\rangle^k$
More general $\frac{1}{4}$ BPS profiles

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Act with fermion zero modes
More general $\frac{1}{4}$ BPS profiles

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More general class of D1-D5 ground state

$\sim a^2$ copies

$\sim b^2$ copies
More general $\frac{1}{4}$ BPS profiles

Orbifold CFT: $k$ twisted sector

Act with fermion zero modes

More general class of D1-D5 ground state

Holographic dual supertube profile

\[ g_1(v) + ig_2(v) = a e^{2\pi i v/R} \]

\[ \text{"} g_5(v) \text{"} = b \sin(2\pi k v/R) \]

Partitioning of charges:

\[ Q_1 Q_5 = R^2 (a^2 + b^2) \]
Families $\frac{1}{8}$ BPS states in the D1-D5-P system

Generic $\frac{1}{8}$ BPS state: Add general left-moving excitations

Momentum charge, $Q_P = L_{0,\text{left}}$  \hspace{1cm} S = 2\pi \sqrt{Q_1 Q_5 Q_P} \quad \text{(Strominger-Vafa)}
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Very special families of momentum excitations: “Supergraviton gas”

$$\left( | + \frac{1}{2} , + \frac{1}{2} \rangle_1 \right)^{N_0} \otimes \left( \frac{1}{m! n!} (J_+^1)^m (L_{-1} - J_{-1}^3)^n |00 \rangle_k \right)^{N_{k,m,n}}$$

$$N_0 + k N_{k,m,n} = N \equiv N_1 N_5$$
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\[ N_0 + k N_{k,m,n} = N \equiv N_1 N_5 \]

Quantum numbers

Define \( \mathcal{N} = \frac{N_1 N_5}{a^2 + b^2} \)

\[
j_L = \frac{1}{2} \mathcal{N} \left( a^2 + \frac{m}{k} b^2 \right), \quad \tilde{j}_R = \frac{1}{2} \mathcal{N} a^2, \quad N_P = \frac{1}{2} \mathcal{N} \frac{m + n}{k} b^2
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\[ Q_1 Q_5 = R^2 (a^2 + b^2) \]
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$D1$-$D5$ $l+\frac{1}{2},+\frac{1}{2}$ residue

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Quantum numbers

Define $N = \frac{N_1 N_5}{a^2 + b^2}$

$$j_L = \frac{1}{2} N \left( a^2 + \frac{m}{k} b^2 \right) , \quad \tilde{j}_R = \frac{1}{2} N a^2$$

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$D1$-$D5$ $l^{+\frac{1}{2},+\frac{1}{2}}$ residue

Special forms:

Adding pure momentum: $m = 0$.

Vanishing angular momentum: $m = 0, a \to 0$. 

$$Q_1 Q_5 = R^2 (a^2 + b^2)$$
The “Supergraviton gas”

We know the supergravity duals of arbitrary superpositions of states of the form:

\[
\left( | + \frac{1}{2}, + \frac{1}{2} \right)_1 \right)^N \otimes \left[ \otimes \left( \frac{1}{m_i! n_i!} (J^+_1)^{m_i} (L_1 - J^3_1)^{n_i} |00\rangle_{k_i} \right)^{N_{k_i,m_i,n_i}} \right]
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Holographic duals
Add momentum and angular momentum excitations to D1-D5 profiles:

\( g_1(v) + i g_2(v) = a e^{2\pi i v/R} \quad \text{“} g_5(v) \text{”} = b \sin(2\pi k v/R) \)
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to give:
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\tilde{j}_L = \frac{1}{2} \mathcal{N} \left( a^2 + \frac{m}{k} b^2 \right), \quad \tilde{j}_R = \frac{1}{2} \mathcal{N} a^2, \quad N_P = \frac{1}{2} \mathcal{N} \frac{m+n}{k} b^2
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Three mode numbers, \((k,m,n) \Rightarrow\) Supergravity duals depend on:

\[ \chi_{k,m,n} \equiv R^{-1} (m + n) v + \frac{1}{2} (k - 2m) \psi - \frac{1}{2} k \phi \]
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k-mode: \((\psi - \phi) \leftrightarrow j_L = j_R\) responsible for \(j_L = \tilde{j}_R = \frac{1}{2} N a^2\)

m-mode \((v - \psi) \leftrightarrow j_L, N_P\) responsible for \(\tilde{j}_R = \frac{1}{2} N \frac{m}{k} b^2, N_P = \frac{1}{2} N \frac{m}{k} b^2\)
The “Supergraviton gas”

We know the supergravity duals of arbitrary superpositions of states of the form:

\[(| + \frac{1}{2}, + \frac{1}{2}\rangle_1)^{N_0} \otimes \bigotimes_{k_i, m_i, n_i} \left( \frac{1}{m_i! n_i!} (J_{-1}^+)^{m_i} (L_{-1} - J_{-1}^3)^{n_i} |00\rangle_{k_i} \right)^{N_{k_i, m_i, n_i}} \]

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k-mode: \((\psi-\phi) \leftrightarrow j_L = j_R\) responsible for \(j_L = \tilde{j}_R = \frac{1}{2} \mathcal{N} a^2\)

\[\tilde{j}_R = \frac{1}{2} \mathcal{N} \frac{m}{k} b^2, \quad N_P = \frac{1}{2} \mathcal{N} \frac{m}{k} b^2\]

m-mode \((v-\psi) \leftrightarrow j_L, N_P\) responsible for \(\tilde{j}_R = \frac{1}{2} \mathcal{N} \frac{m}{k} b^2\)

n-mode \((v) \leftrightarrow N_P\) responsible for \(N_P = \frac{1}{2} \mathcal{N} \frac{n}{k} b^2\)
Building the Fluctuating BPS Microstate Geometries

IIB Supergravity on $T^4$: Supergravity + two (anti-self-dual) tensor multiplets in six-dimensions

**Six-dimensional metric ansatz:**

\[ ds^2_6 = -\frac{2}{\sqrt{P}} (dv + \beta)(du + \omega - \frac{1}{2} Z_3 (dv + \beta)) + \sqrt{P} V^{-1} (d\psi + A)^2 + \sqrt{P} V d\vec{y} \cdot d\vec{y} \]

*(Gutowski, Martelli and Reall)*
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$u = \text{null time}; \ (v, \psi) \ \text{define a double } S^1 \ \text{fibration over a flat } R^3 \ \text{base with coordinates, } y.$
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$u =$ null time; $(v, \psi)$ define a double $S^1$ fibration over a flat $R^3$ base with coordinates, $y$.

The non-trivial homology cycles are defined through the pinching off of the $S^1 \times S^1$ fibration at special points in the $R^3$ base.
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 ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}} (dv + \beta)(du + \omega - \frac{1}{2} Z_3 (dv + \beta)) + \sqrt{\mathcal{P}} V^{-1} (d\psi + A)^2 + \sqrt{\mathcal{P}} V \, dy \cdot dy
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$u =$ null time; $(v, \psi)$ define a double $S^1$ fibration over a flat $R^3$ base with coordinates, $y$.

The scale of everything is set by the “warp factors:” $V$, $P$ and $Z_3$

The non-trivial homology cycles are defined through the pinching off of the $S^1 \times S^1$ fibration at special points in the $R^3$ base.
Building the Fluctuating BPS Microstate Geometries

IIB Supergravity on $T^4$: Supergravity + two (anti-self-dual) tensor multiplets in six-dimensions

Six-dimensional metric ansatz: $(\text{Gutowski, Martelli and Reall})$

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Maxwell Fields

$$G^{(a)} = d\left[ -\frac{1}{2} \frac{\eta^{ab} Z_b}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) \right] + \frac{1}{2} \eta^{ab} *_4 DZ_b + \frac{1}{2} (dv + \beta) \wedge \Theta^{(a)}$$

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**Electric Potentials**

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**Magnetic Fluxes**
The BPS Equations
The BPS Equations

Layer 1: Conditions on Maxwell Fields

\[ \Theta^{(a)} = *_4 \Theta^{(a)} , \quad *_4 D (\partial_v Z_a) = \eta_{ab} D \Theta^{(b)} , \quad D *_4 D Z_a = -\eta_{ab} \Theta^{(b)} \wedge d\beta . \]

\( (Z_a , \Theta^{(a)}) \) depend upon \((r, \theta)\) and

\[ \chi_{k,m,n} \equiv R^{-1} (m + n) v + \frac{1}{2} (k - 2m) \psi - \frac{1}{2} k \phi \]

A homogeneous linear system
The BPS Equations

Layer 1: Conditions on Maxwell Fields  A homogeneous linear system

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General solution known for two-centered geometries!
The BPS Equations

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General solution known for two-centered geometries!

Layer 2: Conditions on Metric pieces

An inhomogeneous linear system

\[
        ds^2_6 = - \frac{2}{\sqrt{P}} \left( dv + \beta \right) \left( du + \omega - \frac{1}{2} Z_3 \left( dv + \beta \right) \right) + \sqrt{P} \, V^{-1} \left( d\psi + A \right)^2 + \sqrt{P} \, V \, d\vec{y} \cdot d\vec{y} \\
        D\omega + *_4 D\omega - Z_3 \, d\beta = Z_a \Theta^{(a)} \\
        *_4 D *_4 \left( \partial_v \omega + \frac{1}{2} D Z_3 \right) = \partial_v^2 P - ( (\partial_v Z_1)(\partial_v Z_2) - \frac{1}{2}(\partial_v Z_4)^2 ) - \frac{1}{4} \eta_{ab} *_4 \Theta^{(a)} \wedge \Theta^{(b)} \\
        (Z_3 , \omega) \) depend upon \((r, \theta)\) and (quadratic) products of harmonics that depend upon

\[ \chi_{k_i,m_i,n_i} = R^{-1} (m_i + n_i) v + \frac{1}{2} (k_i - 2m_i) \psi - \frac{1}{2} k_i \phi \]
The BPS Equations

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Interesting families of particular solutions known. General solution not known.
Linear system of gravitational BPS equations:

**Critical to constructing the holographic duals of a generic superpositions of the states on multiple, independent strands:**

\[(| + \frac{1}{2}, + \frac{1}{2}\rangle_1)^{N_0} \otimes \bigotimes_{k_i, m_i, n_i} \left( \frac{1}{m_i! n_i!} (J_1^+)^{m_i} (L_1 - J_1^3)^{n_i} |00\rangle_{k_i} \right) N_{k_i, m_i, n_i} \]
A Family of Microstate Geometries deep in the Black-Hole Regime

Add pure momentum states

\[ N_0 + N_{1,0,n} = N_1 N_5 \]

\[ (| + \frac{1}{2}, + \frac{1}{2} \rangle_1)^{N_0} \otimes \left( \frac{1}{n!} (L_{-1} - J_{-1}^3)^n |00\rangle_1 \right)^{N_{1,0,n}} \]
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\tilde{j}_L = \tilde{j}_R = \frac{1}{2} \mathcal{N} a^2
\]

\[
N_P = \frac{1}{2} \mathcal{N} \frac{n}{k} b^2
\]

**D1-D5 residue**

All angular momentum

**P excitations**

Angular momentum \(\equiv 0\)
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Can make \( N_P \) large, \( j_L = j_R \rightarrow 0 \)

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Geometry:

\[ \ell_{AdS}^2 = \sqrt{Q_1 Q_5} \]

\[ ds^2_{BTZ} = \ell_{AdS}^2 \left[ \rho^2 (-dt^2 + dy^2) + \frac{d\rho^2}{\rho^2} + \rho_*^2 (dt + dy)^2 \right] \]

Scale of \( S^1 \) stabilizes at \( \rho_* \ell_{AdS} R \)

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Geometry:

**Flat Space**

\[ \text{Flat Space} \]

\[ \text{AdS}_3 \times S^3 \]

\[ \text{BTZ} \times S^3 = \text{AdS}_2 \times S^1 \times S^3 \]

\[ \text{Smooth cap} \]

\[ \ell_{\text{AdS}}^2 = \sqrt{Q_1 Q_5} \]

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Geometry:

Flat Space \( \rightarrow \) AdS \(_3 \times S^3 \)

AdS \(_3 \times S^3 \) \( \rightarrow \) BTZ \( \times S^3 \) = AdS \(_2 \times S^1 \times S^3 \)

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Scale of \( S^1 \) stabilizes at \( \rho_* \ell_{\text{AdS}} R \)

As \( a \to 0 \): \( j_L = j_R \to 0 \)

Depth of AdS \(_2 \) throat \( \rightarrow \infty \)
Several significant results
Several significant results

• First deep, scaling microstate geometry in Black-Hole regime with $j_L = j_R \rightarrow 0$

• Deep, scaling microstate geometry that goes to BTZ

• Deep, scaling $\Rightarrow$ Arbitrarily large red-shifts
  Microstate Geometry $\Rightarrow$ Smooth cap-off

• Momentum excitations localize at the bottom of the BTZ throat

• Holographic dictionary in $\text{AdS}_3$ for deep $\text{AdS}_2$/BTZ throat

• Geometry dual to states counted by Strominger-Vafa
Microstate Geometries for MSW Black Holes

Phase dependence of fluctuations:

$$\chi_{k,m,n} \equiv R^{-1} (m + n) \nu + \frac{1}{2} (k - 2m) \psi - \frac{1}{2} k \phi$$

AdS$_3$ \((u, v, r)\) 

S$^3$ \((\theta, \psi, \phi)\)
Microstate Geometries for MSW Black Holes

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\[ \Rightarrow \text{Kill all the interesting modes} \]

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Before doing this: first enrich the family of solutions

It is relatively easy to generalize the entire IIB construction to include a KKM dipole charge, \( \kappa \), to the D1-D5 system
### Some T-dualities

**Starting configuration**

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**T-dualize 3 times to IIA:**

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**Uplift to M theory**

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M-theory background

D1-D5-KKM solution ➔ M5-M5-M5 charges: \((Q_1, Q_5, \kappa)\)

+ dipolar/dissolved M2-M2-M2 charges

Dualities + compactification on \(\psi\) lattice:

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Add momentum along common circle (5) … untouched in duality

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→ *Momentum excitations of MSW string wrapping (5) direction* ..
MSW string vs M5 on $\mathbb{T}^6$ (or $\text{K3} \times \mathbb{T}^2$)

- **MSW:** Single M brane wrapped on very ample divisor of CY$_3$
- **Here:** Multiple, disjoint M branes $\mathbb{T}^4$'s in $\mathbb{T}^6$
**MSW string vs M5 on T^6 (or K3 × T^2)**

- **MSW**: Single M brane wrapped on very ample divisor of CY₃
- **Here**: Multiple, disjoint M branes T⁴’s in T^6

• Non-trivial fluctuations require turning deforming Kahler moduli of the tori, “bending” disjoint M5’s into one another …

**Universality of the five-dimensional solution:**

• We have reduced to five-dimensions and so our solution is valid for any Calabi-Yau compactification with the same set of M5-brane charges
Fluctuating Microstate Geometries for MSW Strings

Previous picture compactified on Hopf fiber of $S^3$. 

$BTZ \times S^2 = \text{AdS}_2 \times S^2 \times S^1$
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$\text{AdS}_3 \text{ or } S^1$  $S^2$  $\text{BTZ} \times S^2 = \text{AdS}_2 \times S^2 \times S^1$
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AdS$_3$ or $S^1$ \quad $S^2$

Deep scaling, microstate geometries for momentum excitations of MSW string …

Deconstruction: Attempts to realize black-hole microstate structure with perturbative/singular D0 branes or perturbative momenta on “Deconstructed” MSW string

Here: Precise, fully back-reacted, capped-off BTZ $\times S^2$ realization of the deconstructed configurations …

….. related to D1-D5-P microstate structure
Conclusions
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- We have explicit microstate geometries that are holographic duals to precise families of D1-D5-P CFT states
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- First deep, scaling microstate geometry in *Black-Hole Regime* with $j_L = j_R \rightarrow 0$
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Open issues

- Twisted sector excitations. Relation to multi-centered geometries?

- Holography/CFT states of MSW string dual to new microstate geometries

- Probe the IR physics/large-t correlators of these new geometries