Ungauging a Finite Non-Abelian Group in Two Dimensions

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1. This story can be generalized to finite non-abelian groups.
2. The generalization forces us to reconsider our definition of finite global symmetries.
3. Accordingly, a finite global symmetry of a 2d QFT is in general given by a **fusion category** rather than a group.
4. Such generalized finite symmetries are not esoteric. For instance, they can often be obtained by gauging a non-anomalous subgroup of a finite group symmetry.

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The partition function of gauged theory is obtained by

$$Z[\mathfrak{T}/G, \Sigma] = \sum_{\alpha} Z[\mathfrak{T}, \Sigma, \alpha]$$

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Thus, inserting "enough" number of Wilson lines in regular representation forces all the holonomies to be identity and we obtain back the ungauged theory $\mathfrak{T}$.

We can insert enough number of Wilson lines as follows:

1. Choose a triangulation $\mathcal{T}$ of Σ.
2. Insert Wilson lines in regular representation on the edges of the trivalent graph dual to $\mathcal{T}$.

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If $\mathcal{C}$ admits a line operator A with a local operator m that can be inserted at a trivalent junction of A-lines consistently, then $\mathcal{C}$ can be **gauged**. Such a choice of (A, m) is mathematically characterized by a **Frobenius algebra** in category $\mathcal{C}$.

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In other words, gauging $\widehat{\mathcal{C}}_A$ **ungauges** $\mathcal{C}$.

References

Thank you for your attention. Today's talk was based on the following paper with Yuji Tachikawa:

- **Title:** On finite symmetries and their gauging in two dimensions

  Authors: LB, Yuji Tachikawa

  arXiv: 1704.02330

Invitation: I invite those who are interested to see my poster where I describe these arguments more elaborately.