$\mathcal{N} = 2$ vortex strings and their worldsheet theory

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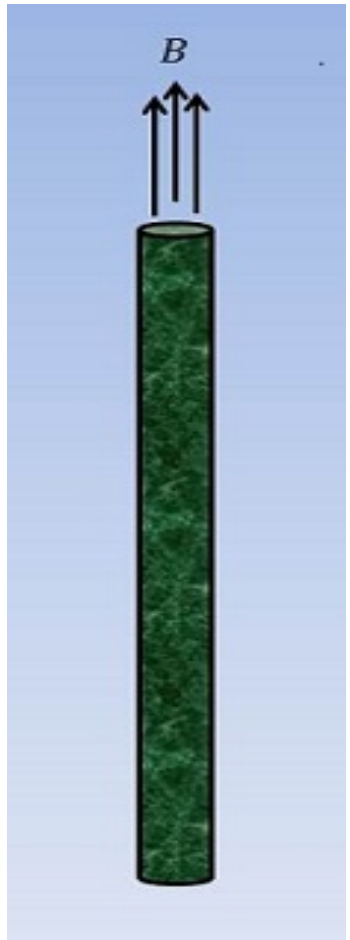
Setup

The theory

Four dimensional $\mathcal{N} = 2$ $SU(N_c) \times U(1)$ gauge theory with N_f hypermultiplets in the fund. of $SU(N_c)$ and charge c_i, $i = 1, ..., N_f$ under the $U(1)$.
When adding a Fayet-Iliopoulos term, the theory contains $\frac{1}{2}$-BPS vortex strings.



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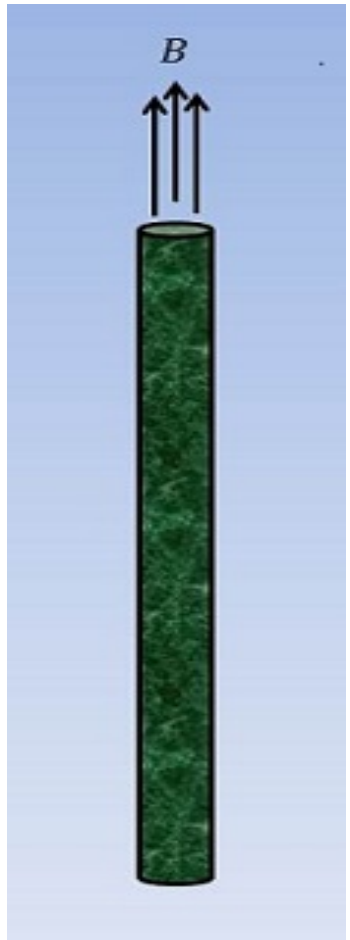

The case of equal $U(1)$ charges ($c_i = 1 \ \forall \ i$) was studied a lot in the literature (Hanany-Tong, Shifman-Yung,...).

- String: Moduli space of solutions.
- Low energy theory: zero modes are promoted to fields on the worldsheet $\Rightarrow$ 2d $\mathcal{N} = (2, 2)$ NLSM.
- 4d-2d map of parameters is weak$\rightarrow$weak: The Kähler class is $t = \tau$.

Goal and motivations

- Goal: Find the worldsheet theory for general N_c, N_f and $U(1)$ charges c_i.
- Wide variety of vacua $\Rightarrow$ wide variety of strings.
- 4d S-duality relates different strings and their worldsheet theories.
- In many cases, the worldsheet theory is strongly coupled $\Rightarrow$ map of parameters is weak$\rightarrow$strong.
  For example, in some theories

$$e^{2\pi i t} = 1 - e^{2\pi i \tau} \;\Rightarrow\; t = O(e^{-S_{inst}}) \,. \tag{1}$$

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When $N_f > N_c$, the vacuum is not gapped. The low energy theory is not two dimensional!
In some cases the 2d and 4d excitations decouple in the IR:

$$S_{eff} = S_{4d} + S_{2d} + irrelevant \ . \tag{2}$$

Residual $\mathbb{Z}_C$ gauge symmetry allows fractional fluxes $\Phi = \frac{2\pi K}{C}$.
Simple quantization condition on $(c_i - c_a)\Phi$:

$\frac{(c_i - c_a)\Phi}{2\pi}$	$\in \mathbb{Z}$	$\notin \mathbb{Z}$
Zero modes decay $\sim \frac{1}{r^p}$	$p \geq 1$	$p < 1$
Decoupling	V	X
AB phases	Trivial	Non-trivial

- Simple criterion for bulk-string decoupling.
- Interesting connection between the decoupling and AB phases.

Tools

- Semiclassical analysis.
- S-duality in some specific cases.
- Localization:

Close the contour and write $\mathcal{Z}_{S^4}$ as a sum over Higgs branch contributions

$$\mathcal{Z}_{S^4} = \sum_K \mathcal{Z}_K \ . \tag{3}$$

Can identify and isolate the string's contribution.
When bulk-string decoupling happens, can write the string's contribution as:

$$\mathcal{Z}_{String} = \mathcal{Z}_{vacuum} \cdot \mathcal{Z}_{S^2} \ . \tag{4}$$

Gives us two things:

1. Non perturbative proof for bulk-string decoupling.
2. Sphere partition function of the worldsheet theory.

Results

- Criterion for bulk-string decoupling.

When decoupling happens, we want to identify the worldsheet theory.

- When the map is weak→weak, we found the worldsheet theories.
- When the map is weak→strong: Found the worldsheet theories in some specific examples ($N_c = 2$ theories which are related by S-duality to weakly coupled theories).
- General case: Have some information about the worldsheet theory (dimension of target space, symmetries, $\mathcal{Z}_{S^2}$, semiclassical insights, ...) but the theory itself is not known yet.