

Vertex Algebras at the Corner

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Introduction

Vertex Operator Algebras (VOA) are algebras of chiral operators in 2d CFT. Some examples are algebras associated to conserved currents J^a , stress-energy tensor T , or W_N algebras generated by

$$J, T, W_3, W_4, \dots, W_N.$$

There exist two constructions of W -algebras:

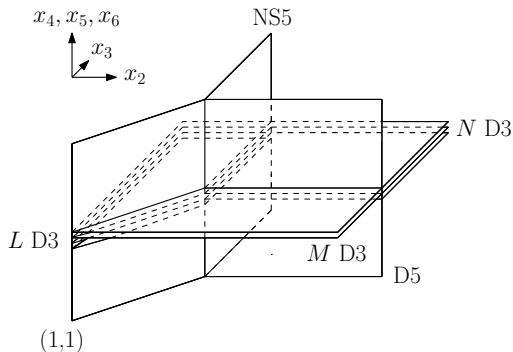
- **Drinfeld-Sokolov (DS) reduction** (labeled as $W[G]$)
- **Coset** (labeled as G/H for $H \subset G$)

Why are there three dual constructions of W_N ?

$$W_N[U(N)_\psi] \leftrightarrow W_N[U(N)_{\frac{1}{\psi}}] \leftrightarrow \frac{U(N)_{\frac{1}{\psi-1}} \times Ff^{U(N)}}{U(N)_{\frac{1}{\psi-1}-1}}$$

Y-algebras

Consider D3-branes attached to the triple junction of D5, NS5, and (1,1) branes in type IIB string theory at each of the three corners.



$$\times C_{x_0, x_1} \times R^3_{x_7, x_8, x_9}$$

From the point of view of D3-branes, this configuration leads to a triple junction of domain walls between $U(L)$, $U(M)$, and $U(N)$ $\mathcal{N} = 4$ super Yang-Mills theories.

Y-algebras

Local operators at the two dimensional corner in the Kapustin-Witten twist of the theory give rise to VOAs. In terms of the notation from the introduction, one can identify the VOA with

$$Y_{L,M,N}[\Psi] = \frac{W_{N-M}[U(N|L)_\Psi]}{U(M|L)_{\Psi^{-1}}}, \quad \text{for } N > M.$$

S_3 **subgroup of the** $SL(2, Z)$ **S-duality group** preserves the trivalent junction and acts as a permutation of the branes accompanied with a transformation of the coupling parameter Ψ . This induces S_3 **duality action on Y-algebras**:

$$Y_{L,M,N}[\Psi] = Y_{N,L,M} \left[\frac{1}{1-\Psi} \right] = Y_{M,N,L} \left[1 - \frac{1}{\Psi} \right]$$

Y-algebras

Examples:

- Three realizations of W_N ($Y_{0,0,N}[\Psi]$):

$$W_N[U(N)_\Psi] \leftrightarrow W_N[U(N)_{\frac{1}{\Psi}}] \leftrightarrow \frac{U(N)_{\frac{1}{\Psi-1}} \times Ff^{U(N)}}{U(N)_{\frac{1}{\Psi-1}-1}}$$

- Three realizations of parafermions ($Y_{0,1,2}[\Psi]$):

$$\frac{U(2)_\Psi}{U(1)_{\Psi-1}} \leftrightarrow \frac{W_2[U(2|1)_{\frac{1}{\Psi}}]}{U(1)_{1-\frac{1}{\Psi}}} \leftrightarrow \frac{U(2|1)_{\frac{1}{\Psi-1}}}{U(2)_{\frac{1}{\Psi-1}-1}}$$

- Many new dualities. . .
- Central charges and vacuum characters agree for all $L, M, N!$

Summary

We have explained that:

- Mysterious triality of W_N is a consequence of S-duality.
- Generalization to four-parameter family of Y-algebras exists.
- Y-algebras can be identified with truncations of W_∞ .

Further developments:

- Generalization to more complicated webs of fivebranes
- Relations to other problems in physics and mathematics such as AGT correspondence, counting of D0-D2-D4 bound states, geometric Langlands program, minimal model holography. . . .