Flows, Fixed Points and Duality in Matter Chern Simons Theory

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Based on

- ArXiv 1707.?????, O. Aharony, S. Jain and S.M.
- 1110.4386 Giombi, S.M. Prakash, Trivedi, Wadia, Yin CS Fermions. Sovability at large N. Non renormalisation of spectrum. Use of lightcone gauge. Exact soln of gap equation and partition function. Proposed bulk dual. First suggestion of bose fermi duality. 1110.4382 Aharony, Gur Ari, Yacobi CS Bosons. Non renormalization of spectrum. Perturbative β function 1112.1016, 1204.3882 Maldacena Zhiboedov Solution for large N correlators using HS sym. 1207.4593 Aharony, Gur Ari, Yacobi Exact solution of 2 and 3 pt functions. Identification of MZ parameters. First concrete duality conjecture and proposal of duality map. 1207.4485 Chang, S.M., Sharma, Yin Vasiliev dual of susy theories including ABJM 1207.4750 Jain, Trivedi, Wadia, Yokoyama Susy thermal partition functions. 1207.4593 Jain, S.M. , Sharma, Takimi, Wadia, Yokoyama Partition function and phase transitions on S^2. Duality of partition functions from level rank duality. 1211.4843 Aharony, Giombi, Gur Ari, Maldacena Yacobi Correct treatment of holonomy in thermal partition function. Duality of thermal partition functions. 1305.7235 Jain, S.M., Yokoyama Flows from susy to quasi fermionic theories 1404.6373, 1505.6371 Jain, Inbasekar, Mandlik, Mazumdar, S.M. Takimi, Wadia, Umesh, Yokoyama Duality of scattering. Modification of crossing symmetry 1507.04378 Gur Ari, Yacoby Re derivation of flows from susy 1511.01902 Radicevic Identification of monopoles as dual Baryons 1512.00161 Aharony Precise conjecture for duality map at finite N and k

Introduction

- Pure $SU(N)$ or $U(N)$ Chern Simons Theory

$$S_{CS} = \frac{k}{4\pi} \int d^3x \, \mathrm{Tr} \left(AdA + \frac{2}{3} A^3 \right) .$$

  Topological. Interacting but exacty solvable.

- Enjoys invariance under nontrivial strong weak coupling level rank duality. $N \leftrightarrow k$. Wilson loops transposed under duality. Rows $\leftrightarrow$ columns. Symmetriic $\leftrightarrow$ Antisymmetric
- Couple matter. Bosons $\leftrightarrow$ fermions? Plenty of evidence the answer is yes.

Quasi Fermionic dualities

- Reg Fermions: $SU(N)_{(k-\frac{1}{2})} + \int \bar{\psi} D_\mu \gamma^\mu \psi$
- Crit Bosons: $U(|k|)_{(-\mathrm{sgn(k)N})} + \int \left(D_\mu \bar{\phi} D^\mu \phi + \sigma \bar{\phi} \phi \right)$
- Duality between Wilson Fisher bosons and Fermions
- Notation

$$N_B = |k|, \quad \kappa_B = -\mathrm{sgn(k)}(N + |k|), \quad \lambda_B = -\mathrm{sgn(k)} \frac{|\mathrm{k}|}{\mathrm{N} + |\mathrm{k}|},$$

$$N_F = N, \quad \kappa_F = \mathrm{sgn(k)(N + |k|)}, \qquad \lambda_\mathrm{F} = \mathrm{sgn(k)} \frac{\mathrm{N}}{\mathrm{N} + |\mathrm{k}|}$$

$$\text{Note} \quad \lambda_B - \lambda_F = \mathrm{sgn}(\lambda_B), \quad \lambda_F - \lambda_B = \mathrm{sgn}(\lambda_F)$$

Quasi Fermionic dualities

- Note theories CFTs. Key point: k integer so gauge coupling $\frac{1}{k}$ cannot run. Mass only relevent operator - fine tuned away, e.g. by using dimensional regularization.
- Calculational evidence for duality in the large N limit includes matching of partition functions, S matrices, spectra of gauge invariant operators, correlators of gauge invariant operators as a function of λ
- In particular the gauge invariant operator $\tilde{J}_0 = \sigma$ maps to $J_0^F = 4\pi\frac{\bar{\psi}\psi}{\kappa_F}$.

Properties of J_0

- $\Delta_{\tilde{J}_0} = 2 + \frac{\delta_B}{N+|k|}$, $\Delta_{J_0^F} = 2 + \frac{\delta_F}{N+|k|}$. δ_B and δ_F unknown. Presume map to each other under duality.
- At leading order in large N

$$\langle \tilde{J}_0(q)\tilde{J}_0(-q')\rangle = (2\pi)^3\delta^3(q-q') \left(\frac{-4\pi|q|}{\kappa_B}\right)\frac{1}{\tan(\frac{\pi\lambda_B}{2})}.$$

$$\langle J_0^F(q)J_0^F(-q')\rangle = (2\pi)^3\delta^3(q-q')\left(\frac{-4\pi|q|}{\kappa_F}\right)\tan\left(\frac{\pi\lambda_F}{2}\right), \tag{1}$$

- Two point functions match under duality

- Three point functions

$$\langle J_0^F(q_1)J_0^F(q_2)J_0^F(q_3)\rangle - \langle \tilde{J}_0(q_1)\tilde{J}_0(q_2)\tilde{J}_0(q_3)\rangle = \frac{6(2\pi)^2}{\kappa_B^2}(2\pi)^3\delta(q_1+q_2+q_3) \tag{2}$$

- Three point functions not equal under duality.

$$\langle J_0^F(q_1)J_0^F(q_2)J_0^F(q_3)\rangle - \langle \tilde{J}_0(q_1)\tilde{J}_0(q_2)\tilde{J}_0(q_3)\rangle = \frac{6(2\pi)^2}{\kappa_B^2}(2\pi)^3\delta(q_1+q_2+q_3) \tag{3}$$

Difference is a contact term, so ok. Implies

$$\int D\phi D\sigma e^{-S_{cb}(\phi,\sigma)+\int \tilde{J}_0\zeta - \frac{(2\pi)^2}{\kappa_B^2}\int \zeta^3(x)} = \int D\psi e^{-S_{rf}(\psi)+\int J_0^F\zeta}. \tag{4}$$

Generalizations?

- Recall ungauged critical boson theory is the end point of an RG flow starting at the regular boson theory
$$\int \left(D_\mu \bar{\phi} D^\mu \phi + \frac{1}{2N_B} b_4 (\bar{\phi}\phi)^2 \right).$$
- Is the same true for the gauged critical boson theory? If so, can the duality of end points be uplifted to a duality of the entire flow?
- Address the first question first. Naively look for RG flows starting at the regular boson theory
$$CS + \int \left(D_\mu \bar{\phi} D^\mu \phi \right) \tag{5}$$
- However, a complication. (5) is not a CFT: consequnce of flow of ϕ^6. To answer questions about flows originating in a CFT, need to understand these flows. We will now proceed to do this.

Quasi Bosonic Theories

- Define

$$S_{RB}(\phi,\sigma,\zeta) = S_{CB}(\phi,\sigma) - \int \tilde{J}_0(x)\zeta(x) + \frac{(2\pi)^2}{\kappa_B^2}\left(x_6^B + 1\right)\int \zeta^3(x). \tag{6}$$

- Similarly define

$$S_{CF}(\psi,\zeta) = S_{RF}(\psi) - \int J_0^F(x)\zeta(x) + \frac{(2\pi)^2}{\tilde{\kappa}_F^2} x_6^F \int \zeta^3(x) \tag{7}$$

- Duality of CB and RF ensures duality of RB and CF as defined. The shift of unity in the coefficient of ζ^3 is due to difference in bosonic and fermionic contact terms.

- Definition of, e.g. the regular fermion theory sound formal and nonlocal. However the path integral over σ in (6) sets $\zeta = \bar{\phi}\phi$ and (6) reduces to

$$S_{RB}(\phi) = CS + + \int \left(D_\mu \bar{\phi} D^\mu \phi + \frac{(2\pi)^2}{\kappa_B^2} \left(x_6^B + 1 \right) (\bar{\phi}\phi)^3 \right) \tag{8}$$

- So S_{RB} is the same theory as (5) except that we allow for a ϕ^6 term- generated whether we want it or not. In particular regular local QFT. To understand the nature of the theory, we must understand how x_6 flows. We turn to this question.

Computation of beta function: Method

$$\int D\phi D\sigma D\zeta e^{-S_{RB}(\phi,\sigma,\zeta)} =$$
$$\int D\phi D\sigma D\zeta e^{-\left(S_{CB}(\phi,\sigma)-\int \tilde{J}_0(x)\zeta(x)+\frac{(2\pi)^2}{\kappa_B^2}(x_6^B+1)\int \zeta^3(x).\right)}=$$
$$\int D\zeta e^{-S_{eff}(\zeta)} \tag{9}$$
$$S_{RB}^{eff}(\zeta) = -\sum_n \int \frac{G_n\zeta^n}{n!} + \frac{(2\pi)^2}{\tilde{\kappa}_F^2}(x_6^B+1)\int \zeta^3(x)$$
$$G_n\zeta^n = \int dp_n \langle \tilde{J}_0(-p_1)\ldots\tilde{J}_0(-p_n)\rangle_{CB}\zeta(p_1)\ldots\zeta(p_n)$$

Effective field theory of field ζ. Nonlocal. But quite explicit. Use to compute S_{RB}^{1PI}. We are interested in $\ln\Lambda$ dependence of S_{RB}^{1PI} at leading nontrivial order in $\frac{1}{N}$

Computation of β function

- Note the coefficient functions, G_n, in $S^{eff}(\zeta)$ are conformal correlators of σ. Λ dependence in correlators completely controlled by $\frac{\delta_B}{\kappa_B}$ the anomalous dimension of σ. Note anomalous dimension $\mathcal{O}(1/N)$. So S^{eff} finite at leading order. Structure of $\ln \Lambda$ depdendence understood at first subleading order.
- $S^{1PI}(\zeta)$ has additional divergences from loops. In a natural normalization $S^{eff}(\zeta)$ has an overall factor of N. Follows that $\frac{1}{N}$ is a loop counting parameter. To work accurately to order $\frac{1}{N}$ we need keep only one loop graphs.
- Need calculate divergent contributions to quadratic and cubic terms in S^{1PI}.
- Once we have the IPI eff action we first rescale ζ to get rid of $\ln \Lambda$ dependence from quadratic term then x_6 to eliminate $\ln \Lambda$ dependence in cubic term. Nature of redefinition determines β function

$$
\begin{aligned}
S^{eff}_{RB} = {} & \frac{g_2}{2\kappa_B}\int dq_2 |q| \left(1 + \frac{2\delta_B(\lambda_B)}{\kappa_B}\ln\frac{|q|}{\Lambda}\right)\zeta(q)\zeta(-q) \\
& + \frac{g_3}{6\kappa_B^2}\int dq_3 \zeta(q_1)\zeta(q_2)\zeta(q_3) \\
& + \frac{\tilde{g}_3}{6\kappa_B^2}\int dq_3 \frac{\delta_B}{|\kappa_B|}\ln\left(\frac{\Lambda}{q_1+q_2+q_3}\right)\zeta(q_1)\zeta(q_2)\zeta(q_3) \\
& - \frac{1}{24\kappa_B^3}\int dq_4 G^0_4(q_1,q_2,q_3,q_4)\zeta(q_1)\zeta(q_2)\zeta(q_3)\zeta(q_4) \\
& - \frac{1}{5!\kappa_B^4}\int dq_5 G^0_5(q_1,q_2,q_3,q_4,q_5)\zeta(q_1)\zeta(q_2)\zeta(q_3)\zeta(q_4)\zeta(q_5),
\end{aligned}
\tag{10}
$$

- Where

$$
\begin{aligned}
g_2 &= \left(\frac{4\pi}{\tan(\frac{\pi\lambda_B}{2})}\right) = -\left(\frac{4\pi}{\cot(\frac{\pi\lambda_F}{2})}\right), \\
g_3 &= (24\pi^2)\left(x_6^B - \frac{4}{3}\cot^2\left(\frac{\pi\lambda_B}{2}\right)\right) \\
&= (24\pi^2)\left(x_6^F - \frac{4}{3}\tan^2\left(\frac{\pi\lambda_F}{2}\right)\right),
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\tilde{g}_3 &= -3r_B(\lambda_B)32\pi^2\left(\frac{\cot^2(\frac{\pi\lambda_B}{2})}{\cos^2(\frac{\pi\lambda_B}{2})} - \frac{1}{4}\right) \\
&= -3\times 32\pi^2\left(\frac{3}{4} + r_F(\lambda_F)\tan^2\left(\frac{\pi\lambda_F}{2}\right)\right) \\
\tilde{G}_4^0(p,-p,k,-k) &= \frac{1}{|p|}\left(\tilde{g}_{(4,1)} + \tilde{g}_{(4,2)}\frac{|k|}{|p|} + \tilde{g}_{(4,3)}\frac{(p\cdot k)^2}{|p|^3|k|}\right), \\
\tilde{G}_5^0(p,-p,0,0,0) &= \frac{1}{p^2}\tilde{g}_{(5,0)}.
\end{aligned} \tag{12}
$$

Beta function diagrams

The diagrams that contribute to S^{1PI} at quadratic order are

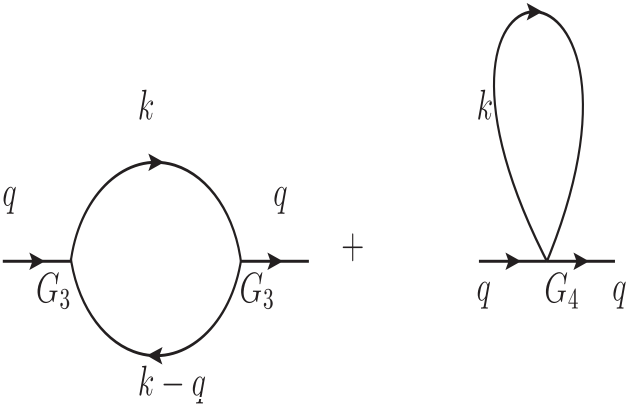


Figure : : First diagram quadratic in x_6 but finite. Second diagram indep of x_6. Has $\ln \Lambda$ divergence.

The diagrams that contribute to the cubic part of S^{1PI} are

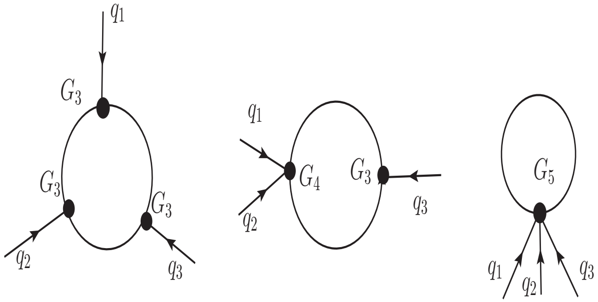


Figure : All diagrams have $\ln \Lambda$ divergence. 1st cubic in x_6. Second linear in x_6. Third indep of x_6.

Tests of resultant IPI effective Action

- Correctly reproduces known anomalous dimension of σ at order $\frac{1}{N}$ at $\lambda_B = 0$.
- Correctly reproduces known anomalous dimension of the Gross Neveu fermions at order $\frac{1}{N}$ at $\lambda_F = 0$.
- Correctly reproduces full β function of x_6 - previously computed by Pisarski - at $\lambda_B = 0$.

Beta Function, result

$$
\begin{aligned}
\frac{dx_6}{d\ln\Lambda} &= -a + by - cy^3 \\
y &= (24\pi^2)\left(x_6 - \frac{4}{3}\cot^2\left(\frac{\pi\lambda_B}{2}\right)\right) = (24\pi^2)\left(x_6 - \frac{4}{3}\tan^2\left(\frac{\pi\lambda_F}{2}\right)\right), \\
c &= \left(\frac{1}{2\pi^2\kappa_B g_2^3}\right) > 0
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
a &= -\left(\frac{\tilde{g}_{(5,0)}}{4\pi^2\kappa_B g_2} - \frac{\tilde{g}_3\delta_B}{|\kappa_B|}\right), \\
b &= -\left(\frac{3\tilde{g}_{(4,1)}}{4\pi^2\kappa_B g_2^2} + \frac{3\delta'_B}{\kappa_B}\right),
\end{aligned}
\tag{14}
$$

a and b known explicitly only at small λ_B and λ_F. β function has 3 zeroes if

$$27a^2c < 4b^3$$

Flows of x_6

The RG flows of x_6 can be plotted as follows

Figure : The points 2 and 1 coincide at $\lambda_B = 0$. They split up at small λ_B. At $\lambda_F = 0$, the point 2 is exactly centred between 1 and 3.

Summary of first part

- Let us summarize. We have demonstrated the β function for x_6 is an exactly cubic function of x_6 at leading order in large N. We have either 3 or one fixed points at every value of λ_B. Consideration of the limits suggests - and we conjecture - that there are 3 FPs at every $\lambda_B > 0$. FP 2 has 2 relevant operators while FPs 1 and 3 have 3 relevant operators.
- The β function computed above is manifestly duality invariant - and so applies to both the Chern Simons coupled regular boson and critical fermion theories, which will thus be dual to each other atleast at large N. We will now argue that this duality persists at large but finite N.

Flows from Susy

- Consider the superconformal $SU(N)$ Chern Simons theory with 4 supercharges that has a single fundamental chiral multiplet. The theory has 3 relevant operators and so gives rise to a 2 parameter set of RG flows.
- 4 years ago I studied these effectively computable RG flows in the large N limit with Jain and Yokoyama. We found that the generic flows lead to theories with a mass gap. However a one parameter tuning of these flows keeps them critical. The critical flows generically end up in either the CB or RF theories.
- The finite N and k self duality of the susy theories maps flows that end up in the CB theory to flows that end in the RF theory, generating the dualities.
- We have checked that a further fine tuning of this one parameter set of theories yields special flows that end up on the 'manifold' of RB or CF theories.

Critical Flows from the Susy theory

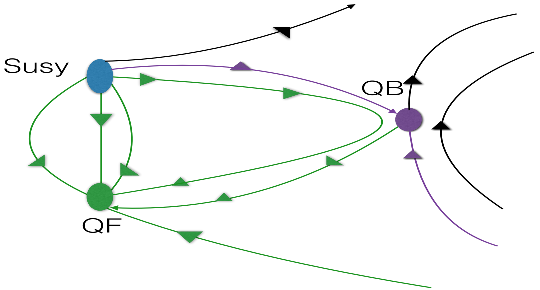
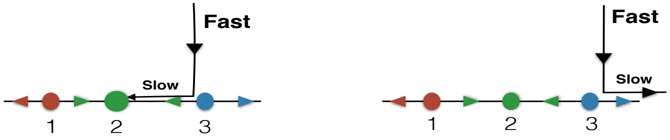


Figure : Generic flows from the susy theory end up at CB or RF theories. Tuned flows end up at RB or CF line of theories. Note these flows - governed by β functions or order unity -are fast. The fast flows end up at a given value of x_6. x_6 then undergoes a slow flow.

Subsequent slow flow

- It turns out that for $\lambda_B < \lambda_c$ the fast flow ends up at a value of x_6 between points 2 and 3. The subsequent slow flow ends up at point 2. For $\lambda > \lambda_c$ the fast flow ends up at a value of x_6 larger than point 3. The subsequent flow ends up at the CB and RF theories.

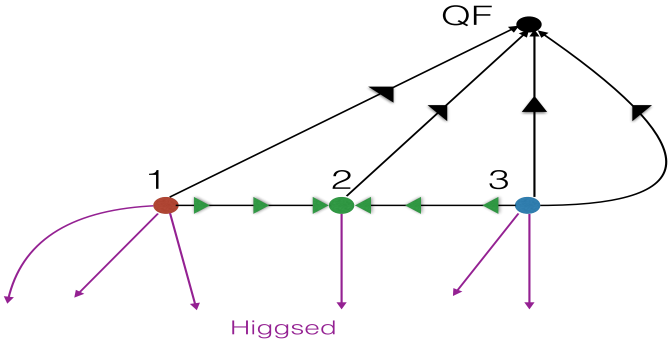


Duality at finite N

- Atleast for a range of λ dual pairs of susy flows end up in the CB and RF fixed points 2.
- This strongly suggests that the duality between (at least) the CB and RF fixed points 2 persists at large but finite N.
- The argument goes as follows. The difference between the finite N and large N beta functions is small. As a consequence the large N flows that were tuned to hit the quasi bosonic manifold miss it by only a small amount, and can always be retuned to hit. Imp point: as many parameters in SUSY flows as relevant operators about the quasi fermionic manifold.

Flows to CB and RF theories

- As the last diagram indicated there exist RG flows from the quasi bosonic to quasi fermionic theories.



Conclusions

- There have been many suggestions that there should be some sense in which Chern Simons coupled regular bosons are dual to Chern Simons coupled critical fermions. We have given this conjecture a precise shape.
- In particular we have argued for the existence of 3 new regular bosonic and 3 new critical fermionic Chern Simons matter fixed points at every large enough value of N and k. We conjecture the new bosonic and fermionic fixed points are dual to each other.
- These new dualities are more general than our old dualities. New imply old under RG flow: other way not true.
- What is this good for? I really dont know. Strings 2023?